

BRIDGE COURSE FOR UG SEMESTER III PAPER - I **REPRODUCTIVE BIOLOGY OF ANGIOSPERMS** AND PLANT GROWTH AND DEVELOPMENT

GF

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Reproduction in Lower and Higher Plants

Can you recall?

- 1. How do plants reproduce without seeds?
- 2. How does vegetative propagation occur in nature ?

Reproduction is the production of young ones like parents. Reproduction is an essential process as it leads to continuation of species as well as to maintain the continuity of life. Each organism has its own particular method of reproduction. All these methods generally fall into two categories :

- i. Asexual reproduction
- ii. Sexual reproduction.

1.1 Asexual Reproduction :

Asexual reproduction does not involve fusion of two compatible gametes or sex cells. It is the process resulting in the production of genetically identical progeny from a single organism and inherits the genes of the parent. Such morphologically and genetically identical individuals are called clones. Organisms choose to reproduce asexually by different modes or ways:

i. Fragmentation : Multicellular organisms can break into fragments due to one or the other reasons. e.g. Spirogyra. These fragments grow into new individuals.





- **Budding**: It is the most common method ii. of asexual reproduction in unicellular Yeast. Usually it takes place during favourable conditions by producing one or more outgrowths (buds). These buds on seperation develop into new individual.
- iii. Spore formation : In Chlamydomonas asexual reproduction occurs by flagellated, motile zoospores which can grow independently into new individuals.



Fig. 1.2 : Zoospores in Chlamydomonas

Other methods of asexual reproduction include - Binary fission which occurs in Amoeba, Paramoecium; Conidia formation in *Penicillium* and **Gemmules formation** in Sponges.

Activity:

Sprinkle a small spoonful of yeast over a warm water and then add sugar. Cover it and wait for 10 minutes. Yeast becomes bubbly over the water proving that it is still active.

Can you recall?

The capacity to reproduce by vegetative propagation :

- Root Sweet potato, Asparagus, Dahlia.
- Leaf Bryophyllum, Kalanchoe, Begonia, etc.
- Stem rhizome (turmeric), tubers (potato), bulbs (onion), etc.
- How does vegetative propagation occur in nature?

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1

Vegetative Reproduction :

Plants reproduce asexually through their vegetative parts. Hence, the new plants formed are genetically identical to their parents.

There are also few methods which would not occur naturally in the plants. Agriculture and horticulture exploit vegetative reproduction in order to multiply fresh stocks of plants. Artificial methods are used to propagate desired varieties according to human requirements. The various methods are as follows :

a. Cutting :

The small piece of any vegetative part of a plant having one or more buds is used for propagation *viz*. Stem cutting - e.g. Rose, *Bougainvillea*; leaf cutting - e.g. *Sansvieria*; root cutting e.g. Blackberry.

b. Grafting :

Here parts of two plants are joined in such a way that they grow as one plant. In this method, part of the stem containing more than one bud (**Scion**) is joined onto a rooted plant called **stock**, is called grafting. Whereas budding is also called **bud grafting** in which only one bud is joined on the stock, e.g. Apple, Pear, Rose, etc.



Fig. 1.3 : Grafting in Rose

c. Tissue culture : It is a method by which a small amount of plant tissue is

Do you know ?

Why does gardner choose to propagate plants asexually?

carefully grown to give many plant lets. Micropropagation method is also used now a days.

1.2 Sexual Reproduction :

It involves fusion of two compatible gametes or sex cells. All organisms reach to the maturity in their life before they can reproduce sexually. In plants, the end of juvenile or vegetative phase marks the begining of the reproductive phase and can be seen easily in the higher plants at the time of flowering.

The flower is specialized reproductive structure of a plant in which sexual reproduction takes place. The function of flower is to produce haploid gametes and to ensure that fertilization will take place. Typical flower consists of four different whorls viz. calyx, corolla androecium and gynoecium.

Sexual reproduction involves two major events viz. meiosis and fusion of gametes to form diploid zygote and the production of genetically dissimilar offsprings. Variations are useful from the point of view of the survival and the evolution of species, over the time.

Sexual reproduction is characterised by fusion of the male and female gametes (fertilization), the formation of zygote and embryogenesis. Sequential events that occur in sexual reproduction are grouped into three distinct stages *viz*, Pre-fertilization, Fertilization and the Post-fertilization.





Always Remember

Diploid sporophyte is the predominant plant body in all angiosperms, where meiosis takes place to produce haploid spores that form gametophyte. Gametophytes are considerably reduced and develop within the flower. They produce gametes.

The male reproductive whorl of flower is called **androecium**. Individual member of androecium, is called **stamen**. Stamen consists of filament, connective and anther.

Structure of Anther :

An immature stage of anther is represented by group of parenchymatous tissue surrounded by single layered epidermis. Anther is generally dithecous (having two lobes) and tetrasporongiate. Each monothecous anther contains two **pollen sacs**. In dithecous anther four pollen sacs are present. Therefore, it is **tetrasporongiate**. The heterogenesity (differenciation) arises when some hypodermal cells get transformed into **archesporial cells**.

T. S. of Anther :

The archesporial cell divides into an inner sporogenous cell and outer primary parietal cell. Sporogenous cell forms sporogenous tissue. Each cell of sporogenous tissue is capable of giving rise to a microspore tetrad. Parietal cell undergoes divisions to form anther wall layers. The wall of mature anther consists of four layers. Epidermis is the outermost protective layer made up of tabular (flattened) cells. Endothecium is sub-epidermal layer made up of radially elongated cells with fibrous thickenings. Inner to endothecium is middle layer made up of thin walled cells (1-2 layered), which may disintegrate in mature anther. Tapetum is the inner most nutritive layer of anther wall. It immediately encloses the sporogenous tissue (microspore mother cells).



Fig. 1.4 : (a) T. S. of anther



Fig. 1.4 : (b) Dehisced anther

1.3 Microsporogenesis :

Each microspore mother cell divides meiotically to form tetrad of haploid microspores (**pollen grains**).

Structure of microspore :

Typical pollen grain is a non-motile, haploid, unicellular body with single nucleus. It is surrounded by a two layered wall called **sporoderm**. The outer layer **exine** is thick and made up of complex, non-biodegradable, substance called **sporopollenin**. It may be smooth or with a sculptured pattern (characteristic of the species). It is resistant to chemicals. At some places exine is very thin showing thin areas known as **germ-pores**. These are meant for the growth of emerging pollen tube during germination of pollen grain. The inner wall layer, **intine** consists of cellulose and pectin.

Find out

Why pollen grains can remain well preserved as fossil?

3

Always Remember

Pollen viability (viability is the functional ablity of pollen grain to germinate to develop male gametophyte) depends upon environmental conditions of temperature and humidity. It is 30 minutes in rice and wheat. But in some members of family Solanaceae, Rosaceae, Leguminosae, it lasts even for months.

Development of male gametophyte :

Pollen grain marks the begining of male gametophyte. It undergoes first mitotic division to produce bigger, naked **vegetative cell** and small, thin walled **generative cell**. The vegetative cell is rich in food and having irregular shaped nucleus. The generative cell floats in the cytoplasm of vegetative cell.





Fig. 1.5 : Development of male gametophyte

The second mitotic division is concerned with generative cell only and gives rise to two non-motile male gametes. The mitotic division of generative cell takes place either in pollen grain or in the pollen tube. The pollen grains are shed from the anther, at this two- celled stage in most of the angiosperms. Female reproductive whorl of flower is gynoecium (Pistil). Individual member of gynoecium is called **carpel** (megasporophyll). A flower with many, free carpels is called **apocarpous** (e.g. *Michelia*). A **syncarpous** flower is one that has many carpels fused together (e.g. Brinjal). Typical carpel has three parts viz, ovary, style and stigma. The number of ovules in the ovary varies e.g. paddy, wheat and mango are **uniovulate** whereas tomato and lady's finger are **multiovulate**.

1.4 Structure of Anatropous ovule :

Each ovule develops inside the ovary and is attached to the **placenta** by a small stalk called **funiculus**. The place of attachment of funiculus with the main body of ovule, is called **hilum**. In angiosperms, the most common type of ovule is **anatropous** in which micropyle is directed downwards and is present adjacent to the funiculus (funicle). The ovule consists of central parenchymatous tissue, the **nucellus** which is surrounded usually by two protective coverings called **integuments** viz. Outer and an inner integument.

A narrow opening at the apex of the ovule is called **micropyle**. Chalaza is the base of ovule directly opposite to micropyle. **Embryo sac** (female gametophyte) is oval multicellular structure embedded in the nucellus.



Fig. 1.6 : Anatropous Ovule





Fig. 1.7 : Development of female gametophyte

1.5 Megasporogenesis :

It is the process of formation of haploid megaspores from diploid **megaspore mother cell** (MMC). Megaspore mother cell becomes distinguished in the nucellus, more or less in the centre but towards micropylar end of ovule.

Development of female gametophyte :

Megaspore mother cell undergoes meiosis to form linear tetrad of haploid cells i.e. megaspore. Upper three megaspores abort and lowest one towards centre of nucellus remains functional. It acts as the first cell of female gametophyte. Generally one megaspore towards centre is functional megaspore. It is infact the first cell of female gametophyte. It undergoes three successive, free nuclear mitotic divisions. Thus total eight nuclei are formed, four of which are located at each pole. One nucleus from each pole migrates towards the centre and are called polar nuclei. Three nuclei towards micropylar end constitute egg apparatus. It consists of large central, haploid egg cell and two supporting haploid synergid cells. Synergid shows hair like projections called filiform apparatus, which guide the pollen tube towards the egg.

Antipodal cells are group of three cells present at the chalazal end. The two haploid polar nuclei of large central cell fuse to form diploid **secondary nucleus** or **definitive nucleus**, just prior to fertilization. This sevencelled and eight nucleated structure is called an **embryo sac**. This method of embryo sac development from a single megaspore is described as **monosporic development**. In angiosperms, the development of female gametophyte is endosporous i.e. within the megaspore. Female gametophyte is colourless, endosporic and is concealed in the ovule enclosed by ovary.

1.6 Pollination :

Pollen grains being non motile, angiosperms have evolved the strategy to use abiotic agents (wind, water) and biotic agents (birds, insects, snails) to their flowers, feeding the visitors and exploiting their mobility for pollination and also seed dispersal. Pollen grains are non-motile and they are usually carried from flower to flower by means of external agents. Pollination is the transfer of pollen grains from anther to the stigma of the flower. It is the prerequisite for fertilization because both the male and female gametes are non-motile. Moreover gametes are produced at two different sites.



Self pollination is a type of pollination which occurs in a single flower or two flowers on a single plant. It results in inbreeding or selfing. In contrast cross pollination is the transfer of pollen grains from the anther of one flower to the stigma of another flower of different plants of same species. Pollination can be further divided into three types on the basis of source of pollination.

a. Autogamy (self pollination) :

It is a type of pollination in which bisexual flower is pollinated by its own pollen grains. Offsprings are genetically identical to their parents e.g. pea.

Always Remember

- Flowers which use autogamy consist of several adaptations in the structure of a flower to facilitate this process. It occurs without external pollinating agents.
- When flower opens to expose its sex organs, it is called **Chasmogamous**.
- The contrivances (a condition that leads to) favour self pollination are-Bisexuality, Homogamy and Cleistogamy.
- Homogamy : When anther and stigma of a flower become mature at the same time, called **homogamy**.
- Some flowers are self pollinated even before the opening of flower. Such condition is called **cleistogamy**. Underground flowers in some plants which exhibit cleistogamy are never opened e.g. *Commelina benghalensis*.
- Plants like *Viola*, *Commelina* can produce both chasmogamous and cleistogamous flowers on the same plant.

Think about it

Why do some plants have both chasmogamous and cleistogamous flowers ?

b. Geitonogamy :

It is the transfer of pollen grain to a stigma of a different flower produced on the same plant. It is functionally similar to cross pollination as it involves pollinating agents, but it cannot bring about genetic variations and is only of ecological significance e.g. *Cucurbita maxima*. It is similar to antogamy as pollen grains come from same plant.

c. Xenogamy (cross polination/ out breeding):

It is a type of cross pollination when pollen grain of one flower is deposited on the stigma of a flower of different plant belonging to same species, with the help of pollinating agency. It generates genetically varied offsprings.

Majority of flowering plants depend on the transfer of pollen grains. Virtually all seed plants need to be pollinated. Most of the food and fibre crops grown throughout the world, depend upon pollinators for reproduction.

The agents responsible for pollination have been grouped into two main categories :

A. Abiotic agents

B. Biotic agents

A. Abiotic Agents : These are non-living agents which include wind and water.

1. Pollination by wind (Anemophily):

Most of the important crop plants are wind pollinated. These includes wheat, rice, corn, rye, barley and oats. Palms are also wind pollinated.

Adaptations in anemophilous flowers :

- The flowers are small, inconspicuous, colourless, without nectar and fragrance (odour).
- The pollen grains are light in weight, dry and produced in large numbers to increase chances of pollination considering wastage of pollengrains.
- Stigma is feathery to trap pollens carried by wind currents.

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- Stamens are exserted with long filaments and versatile anthers.
- Stamens and stigmas are exposed to air currents.





Always Remember

The pollens of wind pollinated plants are most frequently associated with symptoms of hayfever among people those are sensitive to pollens. It is caused by hypersensitivity to pollen.

2. Pollination by water (Hydrophily) :

Found only in some 30 genera of aquatic monocots. E.g. *Vallisneria*, *Zostera*, *Ceratophyllum* etc.

Adaptations in hydrophilous flowers :

- Flowers are small and inconspicuous.
- Perianth and other floral parts are unwettable.
- Pollen grains are long and unwettable due to presence of mucilage.
- Nectar and fragrance are lacking in flowers.

Hydrophily is of two types -

Hypohydrophily : Pollination occurs below the surface of water. Here the pollen grains are heavier than water, sink down and caught by stigmas of female flowers, e.g. In *Zostera* (sea grass) the pollen grains are long, ribbon like and without exine.

Epihydrophily : The pollen grains float on the water surface and reach the stigma of female flower. e.g. *Vallisneria* is a submerged dioecious, fresh water aquatic plant in which female flowers reach the water surface temporarily to ensure pollination and male flowers float on the surface of water.

- Specific gravity of pollen grain is equal to that of water. That is why they float on surface of water.
- Some aquatic plants are anemophilous e.g. *Potamogeton, Halogaris*, etc.
- Some aquatic plants are entomophilous e.g. Lotus, water hyacinth, waterlily, etc.



Fig. 1.9 : Male and female plant Vallisneria

B. Biotic Agents : It includes living agents. About 80% of plants require the help of other living, moving creatures such as insects, birds, bats, snails to transfer their pollens from one flower to another. These also sustain our ecosystems and produce natural resources by helping plants to reproduce.

1. Pollination by insects (Entomophily) : It occurs in Rose, Jasmine, *Cestrum*, etc.

Adaptations in entomophilous flowers :

- They are large, showy and often brightly coloured.
- The flowers produce sweet odour (smell) and have nectar glands.



- The stigma is rough due to presence of hair or is sticky due to mucilaginous secretion.
- The pollen grains are spiny and surrounded by a yellow sticky substance called pollenkit.
- Some plants have special adaptations for the insect visitor to help in cross pollination, e.g. lever mechanism or turn-pipe mechanism in *Salvia*.

Oo you know ?

In biotic pollination, plants are adapted to encourage the specific pollinators they need. They are said to have developed pollination contrivance. Plants and pollinators have co-evolved physical characteristics that make them to interact successfully. Such characteristics are considered pollination syndromes.



Fig. 1.10 : Lever mechanism in Salvia

Think about it

You may see bumblebee early in the year as they try to find a suitable place to establish a nest and rear a colony. If you find a bumblebee nest please leave it alone. Their nest lasts only for a season. Educate the world about the need to help the bees.

2. Pollination by birds (Ornithophily) :

Only a few types of birds are specialised for pollination. They usually have small size and long beaks e.g. Sun birds and humming birds. Some ornithophilous plants are *Bombax*, *Callistemon* (Bottle Brush), *Butea*, etc.



Fig. 1.11 : Ornithophily

Adaptations in ornithophilous flowers :

- Flowers are usually brightly coloured, large and showy.
- They secrete profuse, dilute nectar.
- Pollen grains are sticky and spiny.
- Flowers are generally without fragrance, as birds have poor sense of smell.

3. Pollination by Bats (Chiropteryphily) :

Bats can transport pollens over long distance, some times several kilometers.

Adaptations in Chiropterphilous flowers :

- Flowers are dull coloured with strong fragrance.
- They secrete abundant nectar.
- Flowers produce large amount of edible pollen grains, e.g. *Anthocephalous* (kadamb tree), *Adansonia* (Baobab tree), *Kigelia* (Sausage tree).

1.7 Outbreeding devices (contrivances):

Many plants have mechanisms that discourage or prevent self pollination. To promote cross pollination and increase genetic diversity, plants have evolved a wide variety of sexual strategies. Genetic diversity is an essential factor for evolution by natural selection. Continued self pollination results in the inbreeding depression.

Thus plants have developed many devices to encourage cross pollination. The examples of outbreeding devices are as follows:

Unisexuality :

In this case, the plant bears either male or female flowers. It is also called as **dioecism**. As flowers are unisexual, self pollination is



not possible. Plants may be monoecious, e.g. Maize or dioecious, e.g. Mulberry, Papaya.

Dichogamy :

In this device, anthers and stigmas mature at different times in a bisexual flower so as to prevent self pollination. It can be further divided into two types:

- **1. Protandry :** In this type, androecium matures earlier than the gynoecium, e.g. in the disc florets of sunflower.
- 2. **Protogyny :** In this type, gynoecium matures earlier than the androecium, e.g. *Gloriosa*.

Prepotency :

Pollen grains of other flowers germinate rapidly over the stigma than the pollen grains from the same flower, e.g. Apple.

Heterostyly (heteromorphy):

In some plants like *Primula* (Primrose, there are two or three forms/ types of flowers in which stigmas and anthers are placed at different levels (heterostyly and heteroanthy). This prevents the pollens from reaching the stigma and pollinating it. In heteromorphic flowers, pollen grains produced from anther pollinate stigmas produced at the same level.

Herkogamy :

It is a mechanical device to prevent self pollination in a bisexual flower. In plants, natural physical barrier is present between two sex organs and avoid contact of pollen with stigma of same flower, e.g. *Calotropis*pentangular stigma is positioned above the level of anthers (pollinia).

Self incompatibility (self sterility):

This is a genetic mechanism due to which the germination of pollen on stigma of the same flower is inhibited, e.g. Tobacco, *Thea*.

Do you know ?

In all breeding programmes, the plants are hand pollinated to ensure cross pollination between selected varieties. e.g. wheat, rice.

1.8 Pollen - Pistil Interaction :

It is the interaction of pollen grains with sporophytic tissue (stigma). It begins with pollination and ends with fertilization. All the events from the deposition of pollen grain on stigma to the entry of pollen tube in the ovule (synergid) are referred as pollen - pistil interaction. Pollination does not guarantee the transfer of right type of pollen, often wrong type also land on stigma. The pistil has the ability to recognise and accept the right or compatible pollen of the same species. Thus wrong type of pollen is discarded by pistil. Compatibility incompatibility of the pollen-pistil is and determined by special proteins. This process involves pollen recognition followed by promotion or inhibition of pollen.

The stigmatic surface of flower refuse other wrong type or incompatible pollen grains. A physiologial mechanism operates to ensure that only intraspecific pollen germinate successfully. The compatible pollen absorbs water and nutrients from the surface of stigma, germinates and produces pollen tube. Its growth through the style is determined by specific chemicals. The stigmatic surface provides the essential prerequisites for a successful germination, which are absent in the pollen. The pollen tube is finally pushed through the ovule and reaches the embryo sac. The tip of the pollen tube enters in one of the synergids and then ruptures to release the contents. Due to pollen pistil interaction, intense competition develops even in the compatible pollen grains (gametes).

It also plays important role in sexual reproduction and seed formation. Pollen grain can also be induced to germinate in a synthetic medium. Sucrose induces pollen germination and tube growth *in vitro*. Addition of boric acid facilitates and accelarates pollen germination.

Artificial hybridization :

It is one of the major approaches used in the crop improvement. Only the desired

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pollen grains are hand pollinated and used for fertilization. This is accomplished through emasculation and bagging procedure.

1.9 Double Fertilization :

Double fertilization is a complex fertilization mechanism in flowering (angiospermic) plants. It was discovered by Nawaschin in the liliaceous plants like *Lilium* and *Fritillaria*.

After a pollen grain has reached the surface of the stigma, it germinates and forms a pollen tube, which penetrates the stigma, style, ovary chamber and then enters ovule. The growth of pollen tube is guided by the chemicals secreted by the synergids. It usually enters ovule through the micropyle. It is termed as **porogamy**. But in some cases, it is found to enter through chalaza, known as **chalazogamy** and in some plants by piercing the integuments, called **mesogamy**. Finally, it penetrates embryo sac of ovule through its micropylar end.

The pollen tube carrying male gametes penetrates in one of the synergids. Watery contents of synergid are absorbed by pollen tube which then ruptures and release the contents, including the two non-motile male gametes. As non motile male gametes are carried through hollow pollen tube, it is known as **siphonogamy** that ensures fertilization to take place. **Syngamy** and **triple fusion** are two events of sexual reproduction in angiospermic flowering plants. Syngamy is the fusion of haploid male gamete with haploid female gamete (egg) to produce a **diploid zygote**, whereas in triple fusion, second haploid male gamete fuses with diploid secondary nucleus producing **primary endosperm nucleus** (PEN) that developes into **triploid endosperm**. The zygote develops into an embryo. Syngamy is a type of generative fertilization whereas triple fusion is a type of vegetative fertilization.

Here, both the male gametes participate and therefore, it is described as or called **double fertilization**.



Fig. 1.13 : Double fertilization

Significance of Double Fertilization :

- It is a unique feature of angiosperms. It ensures that the parent plant invests a seed with a food store, only if the egg is fertilized.
- The diploid zygote develops into an embryo which consequently develops into a new plant.

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- The triploid PEN develops into nutritive endosperm tissue.
- It restores the diploid condition by fusion of haploid male gamete with haploid female gamete (i.e. through syngamy).
- It also helps to avoid polyembryony.

1.10 Development of Endosperm :

The triploid primary endosperm nucleus repeatedly divides, mitotically to form nutritive tissue, called **endosperm**. In post-fertilization changes within the ovule, the embryo and endosperm are seen to develop simultaneously.

The other cells of embryo sac disorganized sooner or later. The formation of triploid endosperm nucleus triggers cell division which leads to the formation of endosperm.



Fig. 1.14 : Types of Endosperm

There are three types of endosperms on the basis of mode of development. These are i. Nuclear type, ii. Cellular type, iii. Helobial type :

a. Nuclear Type :

It is the most common type found in 161 angiospermic families. Here, the primary endosperm nucleus repeatedly divides mitotically without wall formation to produce large number of free nuclei. A big central vacuole appears in the centre of cell pushing the nuclei towards the periphery. Later, walls develop between the nuclei, hence multicellular endosperm is formed. But in several cases cell wall formation remains incomplete. e.g. wheat, sunflower and coconut. Coconut has multicellular endosperm in the outer part and free nuclear as well as vacuolated endosperm in the centre.

b. Cellular Type :

In some plants, division of triploid primary endospermic nucleus is immediately followed by wall formation. So that the endosperm is cellular right from the beginning. It is mostly observed in 72 families of dicots as in members - *Balsam, Petunia, Adoxa,* etc.



Use your brain power

What do you call the kernel that you eat in tender coconut ?

c. Helobial Type :

It occurs in the order Helobiales of monocotyledons. In this case, first divison of primary endosperm nucleus is followed by a transverse wall, which divides the cell unequally. The smaller cell is called **chalazal cell** and larger cell is the **micropylar cell**. Then the nuclei in each cell divide by free nuclear divisions and then walls develop between nuclei in micropylar chamber. It is intermediate between cellular and nuclear type endosperm e.g. *Asphodelus*.

Mosaic Endosperm : Endosperm containing tissues of two different types is called mosaic endosperm. In plants like corn the endosperm contains patches of two different colours. It forms a sort of mosaic pattern.

1.11 Development of Embryo :

The process of development of zygote into an embryo is called **embryogenesis**. The embryo is developed at the micropylar end of embryo sac. The growth of embryo triggers only





celled Oospore. Β. Two Α. proembryo. e=embryonal initial: t=suspensor initial; m=Embryo sac membrane. B1=4-celled I-shaped proembryo; e1, e2 are from embryonal initial; s₁, s₂ are from suspensor initial. C. Further development S=Suspensor, of embryo. h=Hypophysis; E=Embryonal mass D. L. S. of ovule Endo=Endosperm in free nuclear stage. Anti=Antipodal Embryo= tissue. Developing embryo E. Embryo showing further development of embryonic octants and hypophysis. F. L. S. of ovule. Endosperm becoming cellular. G. Embryo Cot=Cotyledons; Hypo=Hypocotyl; Rad=Radicle; R.c=Root-cap H. Mature seed. Pl=Plumule. Endosperm has been consumed almost completely.

Fig. 1.15 : Development of Dicot Embryo as in Capsella

after certain amount of endosperm is formed. After fertilization the embryonic development begins.

The zygote divides to form two- celled **proembryo**. The larger cell towards the micropyle is called basal or **suspensor initial** cell and smaller cell towards chalaza is called terminal or **embryonal initial** cell. The suspensor cell divides transversely in one plane to produce filamentous suspensor of 6-10 cells.

The first cell of the suspensor towards the micropylar end becomes swollen and function as a **haustorium**. The lowermost cell of suspensor is known as **hypophysis**. The suspensor helps in pushing the embryo in the endosperm. The embryonal initial undergoes three successive mitotic divisions to form octant. The planes of divisions are at right angles to each other. The lower tier of four cells of octant give rise to hypocotyl and radicle whereas four cells of



Fig. 1.16 : Development of Monocot Embryo



upper tier form the plumule and the one or two cotyledons. The hypophysis by further division gives rise to the part of radicle and root cap. Subsequently, the cells in the upper tier of octant divide in several planes so as to become heart shaped which then forms two lateral cotyledons and a terminal plumule. Further enlargement of hypocotyl and cotyledons result in a curvature of embryo and it appears horse-shoe shaped.

The embryo development is similar in both dicots and monocots up to the octant stage. The difference appears later. In monocot embryo, single cotyledon occupies terminal position and plumule is lateral. The single shield shaped cotyledon is called as **scutellum**. The protective sheath of plumule is called **coleoptile** and that of radicle is **coleorhiza**. **Finally, ovule is transformed into the seed and ovary into the fruit**.

1.12 Seed and Fruit Development :

The goal of reproduction, in every living organisms including plants, is to create offsprings for the next generation. One of the ways that plants can produce offpsrings is by forming (making) seeds. The flowers must be pollinated in order to produce seeds and fruit. Seed development is initiated by fertilization. The integuments of the fertilized ovule persist and get transformed into the seed coat of mature seed.



Fig. 1.17 : Bean seed (Dicot)



Fig. 1.18 : Maize seed (Monocot)

Seed sometimes consists of two distinct coverings, a typical outer seed coat, the **testa** and the inner thin, membranous **tegmen**.

The nucellus in the ovule may persist in some genera like black pepper and beet as a thin, papery layer, the **perisperm**. In some seeds, the food reserves in the endosperm are partially used up in the development of an embryo. Obviously, in such seeds the endosperm remains conspicuous and fills a greater part of the seed. Thus, the resultant seed is **endospermic** or **albuminous** e.g. Castor, Coconut, Maize, etc.

In other seeds, embryo absorbs food reserve from the endosperm completely during its developmental stages. Thus, endosperm disappears (disorganizes) in mature seeds. The resultant seed is **non-endospermic or exalbuminous** e.g. Pea, bean, etc.

The cotyledons in some non-endospermic seeds act as a food storage and in others they are the first photosynthetic organs. Micropyle persists as a small pore in seed coat to allow the entry of water and oxygen during soaking.

Fruit development is triggered by hormones produced by developing seeds. As mentioned earlier, after fertilization the zygote is formed and the ovary begins to differentiate into the fruit and ovary wall develops into **pericarp**. Pericarp is basically three layered which get differentiated in the fleshy fruit like mango, coconut, etc.



Can you recall?

- 1. What are the parts of the fruit ?
- 2. What is the difference between true fruit and false fruit ?

Significance of seed and fruit formation :

- Fruits provide nourishment to the developing seeds.
- Fruits protect the seeds in immature condition.
- Seeds serve as important propagating organs (units) of plant.
- Seeds and fruits develop special devices for their dispersal and thus help in the distribution of the species.

🕦 Try This

Help to rebuild natural ecosystem. Mix seeds and potting soil together with dry clay. Mould the mixture into small balls and allow them to dry in sun. Throw the same at places suitable for germination.

Dormancy is a state of metabolic arrest that facillitates the survival of organisms during adverse environmental conditions. Structural or physiological adaptive mechanism for survival is called **dormancy**. Mature and viable seeds will not germinate even in the presence of favourable conditions and they are dispersed at different places during dormancy. Viable seeds germinate only after completion of dormancy period.

Think about it

- 1. How long seeds stay viable/ healthy?
- 2. Can old seeds still grow?

Some examples of oldest mature seeds that have grown into viable plants are as follows :

- Lupinus arcticus 10,000 years
- Phoenix dactylifera 2000 years
- Some seeds are short lived, e.g. *Citrus*.
- Some tiny seeds are easy for dispersal. e.g. *Striga*, Orchids, *Orobancha*.

1.13 Apomixis :

It is phenomenon of formation of embryo(s) through asexual method of reproduction without formation of gametes and the act of fertilization. Alternatively, it is unusual sexual reproduction where there is no meiosis and syngamy. Embryo develops in the ovule and ovule developes to form seed .

In apomixis, when a gametophyte organ or cell produces embryo like structure without fertilization, it is termed as **apogamy**. Similarly when diploid sporophyte cell produces a diploid gametophyte without undergoing meiosis is called **apospory**, e.g. Orange, Mango.



Internet my friend

Collect information about seed mother Rahilbai's story. How does she save over 80 varieties of native seeds?

The main categories of apomixis are:

a. Recurrent apomixis :

In this type, the embryo sac generally rise either from an archesporial cell or from some other part of the nucellus. In **diplospory**, the unreduced embryo sac is derived from the diploid megaspore mother cell e.g. *Taraxacum*. In apospory, the nucellar cells give rise to apomictic embryo sac.

b. Non-recurrent apomixis :

In this type, megaspore mother cell undergoes usual meotic division and a haploid embryo sac is formed. Here, the embryo arises either from the egg by parthenogenesis or from some other haploid cells of gametophyte through apogamy. Plants produced by this method are generally sterile and do not reproduce sexually, e.g. *Nicotiana*.

c. Adventive Embryony :

In this type, embryos may develop from somatic nucellus or integuments along with normal **zygotic embryo**. It is common in Mango, Orange, Lemon, etc. It gives rise to a condition called **polyembryony**.



Genetically identical plants can be produced effectively and rapidly by apomixis.

1.14 Parthenocarpy :

This term is coined by Noll (1902). It is the condition in which fruit is developed without the process of fertilization. It occurs naturally in some varities of Pineapple, Banana, Papaya, etc. In these plants, it seems that the placental tissue in the unfertilized ovary produces auxin IAA (Indole-3 Acetic Acid) which is responsible for enlargement of ovary into fruit. The fruit resembles the normally produced fruit but it is seedless.

Use your brain power

What do bananas and figs have in common ?

1.15 Polyembryony :

It is the development of more than one embryos, inside the seed and the condition is described as polyembryony. It was first noticed by Leeuwenhoek (1719) in the seeds of Citrus genus. It is the occurrence of more than one embryo in a seed which consequently results in the emergence of multiple seedlings. The additional embryos result from the differentiation and development of various maternal and zygotic tissues associated with the ovule of seed. Polyembryony may be true or false depending upon whether many embryos arise in the same embryo sac or in different embryo sacs in the same ovule. In adventive polyembryony, an embryo develop directly from the diploid cell of nucellus and integuments as in *Citrus*. In cleavage polyembryony, zygote proembryo sometimes divides (cleaves) into many parts or units. Each unit then developes into an embryo. Polyembryony increases the chances of survival of the new plants. Nucellar adventive polyembryony is of great significance in horticulture.



Think about it

Why are some seeds of *Citrus* referred to as polyembryonic ?

Do you know ?

- 1. Parthenogenesis is the development of embryo directly from egg cell or a male gamete. It is a kind of apogamy.
- 2. Agamospermy : Here plants produce seeds. But embryo, inside it, is produced without (omitting) meiosis and syngamy.
- Parthenocarpy can be induced artificially by - spraying of gibberellins, delaying pollination, use of foreign pollens, etc.
- 4. Genetically uniform parental type seedlings are obtained from nucellar embryos.

🥨 Activity :

Prepare chart for natural vegetative propagation exibited by flowering plants indicating the vegetative part/s and the different examples.

Organ	Part	Name of plant



Q.1 Multiple choice questions.

- 1. Insect pollinated flowers usually posses
 - a. Sticky pollens with rough surface
 - b. Large quantities of pollens
 - c. Dry pollens with smooth surface
 - d. Light coloured pollens
- 2. In ovule, meiosis occurs in
 - a. Integument
 - b. Nucellus

.

- c. Megaspore
- d. Megaspore mother cell
- 3. The ploidy level is NOT the same in
 - a. Integuments and nucellus
 - b. Root tip and shoot tip
 - c. Secondary nucleus and endosperm
 - d. Antipodals and synergids
- 4. Which of the following types require pollinator but result is genetically similar to autogamy?

a. Geitonogamy	b. Xenogamy
c. Apogamy	d. Cleistogamy

- 5. If diploid chromosome number in a flowering plant is 12, then which one of the following will have 6 chromosomes?a. Endospermb. Leaf cells
 - c. Cotyledons d. Synergids
- 6. In angiosperms, endosperm is formed by/ due to
 - a. Free nuclear divisions of megaspore
 - b. polar nuclei
 - c. polar nuclei and male gamete
 - d. synergids and male gamete
- 7. Point out the odd one
 - a. Nucellus b. Embryo sac
 - c. Micropyle d. Pollen grain

Q. 2 Very short answer type questions :

- 1. Name the part of gynoecium that determines the compatible nature of pollen grain.
- 2. How many haploid cells are present in a mature embryo sac ?
- 3. Even though each pollen grain has 2 male gametes, why atleast 20 pollen grains are required to fertilize 20 ovules in a particular carpel?
- 4. Define megasporogenesis.
- 5. What is hydrophily ?
- 6. Name the layer which supplies nourishment to the developing pollen grains.
- 7. Define parthenocarpy.
- 8. Are pollination and fertilization necessary in apomixis ?
- 9. Name the parts of pistil which develop into fruits and seeds.
- 10. What is the function of filiform apparatus ?

Q.3 Short Answer Questions :

- 1. How polyembryony can be commercially exploited ?
- 2. Pollination and seeds formation are very crucial for the fruit formation. Justify the statement.
- 3. Incompatibility is a natural barrier in the fusion of gametes. How will you explain this statement?
- 4. Describe three devices by which cross pollination is encouraged in Angiosperms by avoiding self pollination.

Q. 4 Long Answer Questions :

- 1. Describe the process of double fertilization.
- 2. Explain the stages involved in the maturation of microspore into male gametophyte.



3. Explain the development of dicot embryo.

Q. 5 Fill in the blanks:

The collects the pollen grains.

The male whorl, called the produces

The pollen grains represent the

The contains the egg or ovum.

takes place when one male gamete and the egg fuse together. The fertilised egg grows into seed from which the new plants can grow.

The is the base of the flower to which other floral parts are attached.

Q. 6 Label the parts of seed.



4.	Drc	aw a	labelle	d diagra	am of	the	L.S.
	of	anat	ropous	ovule	and	list	the
	con	npone	ents of e	mbryo s	ac and	d mei	ntion
	thei	ir fate	e after fe	rtilizati	on.		

..... is the transfer of pollen grains from anther of the flower to the stigma of the same or a different flower

Once the pollen reaches the stigma, pollen tube traverses down the to the ovary where fertilisation occurs.

The are coloured to attract the insects that carry the pollen. Some flowers also produce or that attracts insects.

The whorl is green that protects the flower until it opens.

Q. 7 Match the column.

Column - I		Column - II		
(Structure before		(Structure after		
seed formation.		seed formation.		
A.	Funiculus	I.	Hilum	
В.	Scar of Ovule	II.	Tegmen	
C.	Zygote	III.	Testa	
D.	Inner integument	IV.	Stalk of seed	
		V.	Embryo	

a. A - V, B - I, C - II, D - IV b. A - III, B - IV, C - I, D - V c. A - IV, B - I, C - V, D - II d. A - IV, B - V, C - III, D - II

Project :

Natural vegetative propagation by leaves only in different vascular plants.



Hybridization Concept by Dr. Shaukat A, Shah

According to valence bond theory, valency of an element depends upon the number of unpaired electrons present in an atom. In another words number of bonds form by an element is equal to the number of unpaired electrons present in an atom. This can be simplified as follows:

eg,
$$_7N - 1s^22s^2 2p^3$$

Three unpaired electrons, hence nitrogen can form three bonds with other elements

$${}_{8}O - 1s^2 - 2s^2 - 2p^4$$

↑↓ ↑↓	↑↓ ↑	
--------------	------	--

two unpaired electrons hence oxygen can form two bonds

 $_{9}F - 1s^2 - 2s^2 - 2p^5$



One unpaired electron and hence fluorine can form only one bond

Extending to the same rule to carbon

$${}_{6}C-1s^2$$
 $2s^2$ $2p^2$



Two unpaired electrons, hence it should be divalent.

We expect that valency of carbon would be two. This is because carbon contains two unpaired electrons. Thus carbon is expected to form compounds like CH₂, CCl₂ etc. But actually we know that carbon forms compounds like CH₄, CCl₄ etc. Indicating it to be tetravalent and hence

containing four unpaired electrons. In order to explain this, the concept of excitation or promotion is introduced. Hence, is assumed that one of the electron from 2s orbital is excited or promoted to $2p_z$ orbital. This can be indicated as follows:

 ${}_{6}C_{\text{ground state}}$ 1s² 2s² 2p_x¹2p_y¹2p_z⁰ Twounpaired electrons.

 ${}_6C \text{ excited state} 1s^2 2s^1 2p_x{}^{1}2p_y{}^{1}2p_z{}^1$ Now four unpaired electrons.

	♠	₳	♠
--	---	---	---

 p_x ¹Promotion or excitation only. Condition of observed (actual) valency of an element is

explained. To explain the equivalency of bonds is to explain equivalency of bond length, bond angle, bond energy and actual geometrical shape of molecules. It needs mixing of atomic orbitals to form equivalent hybrid orbitals (hybridization).

Formation of Methane (CH₄) sp³ Hybridization

Following steps involved in the formation of CH4 molecule by hybridization process:

Write electronic configuration of central atom i.e. Carbon

$_6C$ ground state	$1s^2$	$2s^2$	2 g	$p_x^{1}2$	$p_y^{1/2}$	$2p_z^0$
	₩	₩	♠	↑		

Write excited state electronic configuration of Carbon to explain observed valency of Carbon:

$${}_{6}C_{\text{excited state}} 1s^{2} 2s^{1} 2p_{x}{}^{1}2p_{y}{}^{1}2p_{z}{}^{1}$$

By writing excited state electronic configuration the condition of tetravalency of carbon is fulfilled.

To explain the equivalence of of bonds and actual geometrical shape of molecules mixing of atomic orbitals containing unpaired electrons is needed.

One 2s orbital and three 2p $(2p_x, 2p_y \text{ and } 2p_z)$ orbitals mixed hence named as sp³ (one 's' character and three p characters).

These four hybrid orbitals are ------at the corners of regular tetrahedron and make an angle 109°28'. This type of hybridization is called as tetrahedral hybridization. Each sp³ hybrid orbital contains one unpaired electron. Formation of methane molecule and sp³ hybrid orbitals are represented as-



In the formation of methane molecule, four sp³ hybrid orbitals of carbon overlap axially with four $1s^1$ atomic orbital of four hydrogen atom which is represented diagrammatically as follows-4H ($1s^1$) + 4 sp³ (hybridised orbitals of carbon)

axial

overlap



Ethene (C_2H_4) sp² hybridization:

In the formation of ethane molecule both carbon undergo sp^2 hybridization. One electron from $2s^2$ orbital is uncoupled and promoted into vacant p orbital of carbon atom. Then one $2s^1$ orbital and two $2p (2p_x^1 \text{ and } 2p_y^1)$ orbitals get mixed and formed three sp^2 hybridised orbitals as shown below:

 $_{6}C:1s^{2}$ $2s^{2}$ $2p_{x}^{1}2p_{y}^{1}2p_{z}^{0}$



Sp2 hybrid orbitals oriented in space as planer to avoid maximum repulsion having bond angle 120°

one $2p_z^1$ orbital of each carbon atom remainunhybridized. Diagrammatical representation of hybridized and unhybridizex orbitals one as shown below:



Sp2-sp2 overlap: sp2 hybrid orbital of one carbon atom overlap with sp2 hybrid orbital of other carbon atom to form C-C sigma bond.

Sp2-s overlap: remaining to sp2 hybridised orbitals of each carbon atom overlap with four 1s1 orbitals of four hydrogen atoms and formed four C-H sigma bond.

Pz-pz overlap: unhybridised pz1 orbital of one carbon atomlaterally overlap with pz1 orbital of another carbon atom to form carbon-carbon (C-C) π bond.

(Note: hybrid orbitals never form π bond.

Geometry of molecule is trigonal planer with bond angle 120°C.

Sp-hybridization in Acetylene (C₂H₂)

In formation of the Acetylene molecule, both carbon atom undergo 'sp' hybridization. One electron from $2s^2$ is promoted to vacant 'pz' orbital of carbon atom and then $2s^1$ and one $2p_x^1$ orbital of carbon get mixed and formed 2sp hybridized orbitals containing single electron each. Two $2p_y^1$ and $2p_y^1$ orbital of both carbon atom remain unhybridised.



sp-sp overlap:

'sp' hybridization orbital of one carbon atom axially overlap with 'sp' hybridization orbital of other carbon atom to form carbon carbon sigma bond (C-C)

sp-s overlap:

sp hybridization orbital of each carbon atom overlap axially with two '1s¹' atomic orbital of two hydrogen atom and formed two C-H sigma bond.

Formation of π bonds between carbon-carbon:

Unhybridizedpy and pz orbitals of one carbon atom laterally overlap with other unhybridized py and pz orbitals of other carbon atom and form two π bonds between carbon-carbon respectively. Hybridized orbitals (sp) oriented in the space linearly to avoid maximum repulsion

Geometry of the molecule is linear with bond angle 180°C

180°C H- $-C \equiv$ $\equiv C$ --н

Nature of covalent bond can be studied if the following parameters are known

Bond length

Bond angle

Bond energy

Bond length: "The average distance between two bonded nuclei in a molecule is called bond length."

Bond length can be measure by X-ray diffraction or spectroscopic measurement.

Н—— Н	0.75 Å
С—С	1.54 Å
С== С	1.34 Å
н—с≡с—н	1.20 Å

Due to stretching vibrations, bond length is not fixed, hence it is taken as average

Bond Angle:

e.g.

"Bond angle is defined as the internal angle between the orbitals containing electrons pairs in the valency shell of the central atom in the molecule."

e.g.



Due to bending vibration, bond angle is also taken as average bond angle.

Bond energy:

The energy required to break one mole of bonds that type in a substance in the gaseous state known as bond energy.

e.g.

с—с	347 kJ/mole
c===c	619 kJ/mole
C≡≡C	836 kJ/mole

Electrophilic and Nucleophilic Reagents:

Electrophiles – Electrophiles are electron deficient species. These reagentsmay be positive ions or neutral molecules containing an atom with incomplete octet. It attacks on the rich electron density site of the substrate. They are lewis acids (electron pair acceptor).

e.g. i) Neutral molecules – BF₃, AlCl₃, FeCl₃ (B, Al and Fe have incomplete octet)

ii) Positive ions $-H^+$, H_3O^+ , NO_2^+ , NH_4^+ , Br^+ , R_3C^+ , $R3C^-$

Nucleophiles – A nucleophile or nucleophilic reagent is negative ion or neutral molecule capable of donating an electron pair. It is lewis base. It attack on the electron deficient site of the substrate.

e.g. i) Negative ions – OH⁻, Cl⁻, Br⁻, I⁻, CN⁻, NH₂⁻, SR⁻, R₃C⁻

ii)Neutral molecules – H₂O, ROH, NH₃, R-NH₂, ROR

Note- The nucleophic or electrophilic sites within the neutral organic molecule can be determined by –

- i) The presence of lone pairs of electrons
- ii) Type of hybridization (sp, sp², & sp³)
- iii) The polarity of the bond.

Session [2018-2019]

Department of Chemistry

Remedial classes

on

Basic concepts in Chemistry

TOPICS

- ***** Atomic structure
- ***** Quantum number
- ***** Electronic Configuration
- **♦** Periodic properties
- ✤ Basic concepts in organic chemistry
- Oxidation and reduction
- * Hybridization



DEPARTMENT OF PHYSICS PAPER - I : ELEMENTS OF MODERN PHYSICS UNIT I : INTRODUCTION TO QUANTUM THEORY

INTRODUCTION:

The development of Classical Mechanics is based on Newton's laws of motion. These laws include the concept of absolute mass, absolute space and absolute time. The Classical Mechanics explains correctly the motion of celestial bodies like planets, stars and terrestrial bodies moving with non relativistic speed (v << c, c: speed of light in vacuum). In addition to this, motion of charged particles in electromagnetic fields, vibrations in string, elastics collisions in solids, propogation of waves in gases, flow of liquid, molecular motion of gases can be explained with the help of classical concepts. Similarly optical phhenomenon such as reflection, refraction, diffraction, scattering, dispersion, polarization, etc. could explain the basis of wave theory of light and electromagnetic theory.

FAILURES OF CLASSICAL MECHANICS ARE AS FOLLOWS:

Classical Mechanics could not explain some phenomenon discovered in the beginning of 20th century:

- Stability of atom
- Observed spectrum of black body radiation
- Photoelectric effect
- Compton effect
- Raman effect etc.

The inadequacy of Classical Mechanics led to the development of Quantum Mechanics.

BLACK BODY RADIATION:

- A body which absorbs radiations of all wavelengths at any temperature is called perfectly black body. When this body heated , at a particular temperature it glows and emits thermal radiation known as Black body radiations.
- As temperature increases the object becomes red then yellow then white.
- The thermal radiation emitted by object consists of continuous distribution of frequency ranging from infrared to ultraviolet.
- The continuous pattern of the distribution spectrum is in sharp contrast to the radiation emitted by heated gas.
- The radiation emitted by gases has a discrete distribution spectrum: a few sharp colored lines with no light in between.

Black Body Spectrum:



- Black Body Spectrum is the graph between wavelength on x-axis and energy density of radiation on y-axis for given black body as shown in fig. above.
- It is clear that the black body does radiate energy at every wavelength above absolute zero.
- The intensity of radiation increases with the wavelength of radiation.

- At particular wavelength (Λ_m) the intensity becomes maximum and beyond Λm the intensity, decreases gradually.
- As the temperature of body is increased, the peak wavelength Λ_m shifts towards the smaller value.
- Also with increase in temperature, the total energy emitted also increases, because the total area under the curve increases.
- The area under curve is directly proportional to the 4th power of absolute temperature of the body i.e. E a T⁴. This is called "Stefan Boltzmann's"

Failure of Classical Mechanics to Explain Black Body Radiation:

There are two laws on the basis of Classical Mechanics to explain Black Body Spectrum

- 1. Wein's Law
- 2. Rayleigh Jeans Law

The laws and their drawbacks are as follows:

1. <u>Wein's Law</u>

According to Wein the product of wavelength (Λ_m) corresponding to the maximum energy and absolute temperature (T) is constant.

$$\lambda_m T = Constant$$

And Wein's displacement law is given by,

$$E_{\lambda}d\lambda = \frac{C_1}{\lambda^5} e^{\frac{-C_2}{\lambda T}} \, d\lambda$$

Where,
$$C_1 \& C_2$$
 are constants

This law explains black body spectrum in shorter wavelength i.e. lower values of ΛT but fails to explain in longer wavelength region i.e. higher values of Λ

2. Rayleigh Jean's Law

Rayleigh and Jean's assumed that a lack body contains some hypothetical oscillator. Which emits waves of all possible frequency and gave the relation,

$$E_{\lambda}d\lambda = \frac{8\pi kT}{\lambda^4}d\lambda$$

This law explains black body spectrum in longer wavelength $\Lambda(\Lambda \times \Lambda)$ region successfully but fail to explain in shorter wavelength Λ region.

PLANCK'S QUANTUM THEORY/ PLANCK'S HYPOTHESIS:

- Planck assumed that the walls of the cavity consist of microscopic oscillators of molecular dimensions.
- The frequency of radiation emitted by oscillator in same as the frequency of its vibration.
- The oscillator cannot emit energy in a continuous manner but in the form of discrete packets of energy. This packet of energy is called as photon.
- Each photon carries an energy which is directly proportional to the frequency of wavelength i.e. E depends upon v.

i.e. E= hv (where v is frequency)

where, $h = 6.634 \times 10^{-34}$ Jsec

<u>Planck's Law</u>

According to Planck average energy of oscillator is,

$$E = \frac{h\nu}{e^{h\nu/_{kT-1}}}$$
-----(1)

The number of oscillations in frequency range v to v+dv is given by,

$$n(\nu)d\nu = \frac{8\pi\nu^2}{c^3}d\nu \quad \text{------(2)}$$

Therefore from 1 & 2, total energy (of oscillator) in frequency range $\upsilon + d\upsilon$ is,

$$E_{\nu}d\nu = E \times n(\nu)d\nu$$

$$E_{\nu}d\nu = \frac{h\nu}{e^{h\nu}/kT-1} \frac{8\pi\nu^{2}}{c^{3}}d\nu$$

$$E_{\nu}d\nu = \frac{1}{e^{h\nu}/kT-1} \frac{8\pi h\nu^{3}}{c^{3}}d\nu$$
(3)
$$\nu = \frac{c}{\lambda} \implies d\nu = \frac{-c}{\lambda^{2}}d\lambda$$

Now,

Therefore, by putting the values of eqn (4) in eqn (3), we get,

$$E_{\lambda}d\lambda = \frac{1}{e^{hc}/_{\lambda kT-1}} \frac{8\pi hc}{\lambda^5} d\lambda -----(5)$$

This is Planck's equation .

(i) In short wavelength region λ is very small i.e. $\frac{hc}{kT}$ is large and hence $e^{hc/_{\lambda kT}}$ much greater than 1 hence is neglected ($e^{hc/_{\lambda kT}} \gg 1$). Hence eqn (5) becomes,

$$E_{\lambda}d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda kT} d\lambda$$
$$E_{\lambda}d\lambda = \frac{C_1}{\lambda^5} e^{-C_2/\lambda T} d\lambda -----(6)$$
where, $8\pi hc = C_1$ and $\frac{hc}{k} = C_2$

Equation (6) is Wein's law.

(ii) In longer wavelength region λ is large i.e. $\frac{hc}{kT}$ is small and $e^{hc}_{\lambda kT} - 1 \approx \frac{hc}{\lambda kT}$. Therefore eqn (5) becomes,

$$E_{\lambda}d\lambda = \frac{\lambda kT}{hc} \frac{8\pi hc}{\lambda^5} d\lambda$$
$$E_{\lambda}d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda -----(7)$$

Equation (7) is Rayleigh Jeans law.

Thus Planck's law completely explains black body spectrum.

PHOTOELECTRIC EFFECT

When the electromagnetic radiation of suitable frequency incident on the metal surface, electrons is emitted from the surface this phenomenon is called as **Photoelectric effect**. The electrons which are emitted in this phenomenon are called **photoelectrons** and the corresponding current is called **photocurrent**.

Experimental arrangement:



- Hertz first observed this effect in 1887.
- The experimental arrangements consist of an evacuated glass tube with a metallic cathode C and a metal anode A. Cathode C and anode A are connected to a variable voltage source along with a galvanometer as shown.
- When the apparatus is kept in dark, galvanometer G shows no current.
- However, when monochromatic light of an appropriate wavelength is made to fall on cathode C, the galvanometer detects a flow of current. The electrons are emitted by cathode C and collected by anode A by moving across the tube between C and A.
- When the voltage V between C and A increases, the current I_p also increases until a saturation stage is reached (a saturation stage is one when all electron emitted by C are collected by A).

The photocurrent depends on:

1. Effect of frequency of light radiation on photocurrent:

For given photosensitive material, photocurrent does not flow below certain frequency called Threshold frequency. The minimum frequency of incident radiations required to eject the electron from metal surface is defined as **threshold frequency** ' v_0 '.

If incident frequency $v < v_0 \Rightarrow$ No photoelectron emission

The maximum wavelength of incident radiations required to eject the electrons from a metallic surface is defined as **threshold wavelength** ' λ_0 '.

If incident wavelength $\lambda > \lambda_0 \Rightarrow$ No photoelectron emission

2. Effect of Intensity of Light Radiation on photocurrent:

For a given frequency $v > v_0$ and constant potential difference, photocurrent increases with increase in intensity of light radiation.

3. Effect of Potential difference on photocurrent:

If anode is at positive potential then photocurrent flows through circuit It increases with increase in anode potential but if anode is at negative potential then photocurrent decreases with increase in anode potential. At particular value of negative potential at anode, photocurrent becomes zero. This anode potential is called stopping potential.

4. Effect of Nature of photosensitive metal on photocurrent:

For the same value of frequency and potential difference, photocurrent is different for different metal. It is found that stopping potential is independent of frequency and dependent of intensity of radiation.

Drawbacks of Classical Theory to explain Photoelectric effect:

- According to wave theory of light, radiation incident on metal is spread out over the metal. Therefore, energy gain by each electron is very small. Hence electron require some time to get sufficient energy to escape from metal thus, photoelectric effect cannot be an instantaneous process but practically this is not so.
- According to CM an oscillating particle with frequency v, can absorbs and emit energy E=hv



Here, the electron is emitting the energy, $E = hv - \phi$ instead of E = hv. Hence failure of Classical Mechanics.

- According to wave theory, photoemission can occur at any frequency of incident radiation but practically photoemission can occur only above threshold frequency.
- According to classical electromagnetic theory, when radiation is incident on metal then electron oscillates. The oscillation depends on intensity of radiation, therefore oscillation and hence energy of electron increases with increasing intensity of radiatiom. But, practically maximum Kinetic Energy of photoelectron is independent of intensity.

EINSTEIN'S PHOTOELCTRIC EQUATION:

According to Einstein, photoelectric effect is the result of one to one inelastic collision between photon and electron in which photon is completely absorbed

Assumption-

• Light consists of bundles of tiny particles called quanta. Each quanta is then called photon. Each photon has energy

Where, h = Planck's constant

v = Frequency of radiation

- When light radiation falls on metal surface then each colliding photon gives its total energy total energy to the electron on which it collides. This energy is used by electron for two purposes.
 - (a) A part of energy (ϕ_0) is used to detach from metal.
 - (b) The remaining part of energy is used to increase its maixmum K.E. = $\frac{1}{2}mv^2$ i.e. $hv = \phi_0 + \frac{1}{2}mv^2$ $\frac{1}{2}mv^2 = hv - \phi_0$

Where, $\phi_0 = hv_0 = \frac{hc}{\lambda}$ is called work function for a given metal.

The minimum energy of incident radiation, required to eject the electrons from metallic surface is defined as **work function** of that surface.

$$\frac{1}{2}mv^{2} = hv - hv_{0}$$

$$\frac{1}{2}mv^{2} = h(v - v_{0}) - \dots - (1)$$

$$\frac{1}{2}mv^{2} = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_{0}}\right)$$

$$\frac{1}{2}mv^{2} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_{0}}\right) - \dots - (2)$$

Equation (1) and (2) are called Einstein's photoelectric equation.

Equation (1) shows that, if

- (a) $\nu < \nu_0$ photoemission does not occur
- (b) $v = v_0$ photoemission does not occur
- (c) $v > v_0$ photoemission does not occur

Einstein's explanation of photoelectric effect:

- According to Einstein's, when light radiation falls on metal surface then each colliding photon gives its total energy to the electron on which it collides and hence, electron is immediately emitted i.e. photoelectric effect is instantaneous process.
- Einstein's photoelectric equation is,

$$\frac{1}{2}mv^2 = h(v - v_0)$$

If $\nu < \nu_0$ then K.E. becomes negative which is not possible hence photoemission does not occur below threshold frequency ν_0 .

 Intensity of radiation is proportional to number of photon per unit time per unit area. If intensity is increases at constant frequency, then only number of photoelectrons emitted per second increases. Thus, the number of photoelectrons is proportional to intensity. Einstein's photoelectric equation is,

$$\frac{1}{2}mv^2 = h(v - v_0)$$

It clearly shows that maximum K.E. of photoelectron is independent of intensity of light radiation.

WAVE PARTICLE DUALITY:

Light has dual nature i.e. it behaves like wave in phenomenon like reflection, diffraction, interference, etc. and it behaves like matter in phenomenon like black body spectrum, photoelectric effect, Compton Effect.

In 1924 de-Broglie suggested that moving matter has properties of waves. Hence matter has dual nature i.e. particle nature and wave nature. The wave nature of matter was explained experimentally by Davisson and Germer

de-Broglie Hypothesis:

<u>Statement</u>: "All the particles in motion have properties of wave. The wavelength of wave associated with particle is inversely proportional to the momentum of particle ."

i.e. $\lambda \propto \frac{1}{P}$

Therefore,

$$\lambda = \frac{h}{P} = \frac{h}{mv}$$
 is called **de-Broglie wavelength**.

Mathematical Proof:

Consider a photon of mass 'm' moving with velocity 'c'.

If we consider photon as a wave of frequency ' ν' , then its energy is,

$$\mathbf{E} = \mathbf{h}\mathbf{v} \quad -----(\mathbf{1})$$

 $E = mc^2$ -----(2)

If we consider photon as a particle, then its energy is,

Now equate equation (1) & (2), we get,

 $h\nu = mc^{2} - \dots - (3)$ $h\nu = pc - \dots - (since, p=mc)$ $\frac{hc}{\lambda} = pc - \dots - (since, \nu = \frac{c}{\lambda})$ $\frac{h}{\lambda} = p \implies \lambda = \frac{h}{p}$ $\implies \lambda \propto \frac{1}{p} - \dots - (4)$

Equation (4) is the equation for de-Broglie wavelength

EXPRESSION FOR DE-BROGLIE WAVELENGTH OF PARTICLE MOVING WITH VELOCITY LESS THAN VELOCITY OF LIGHT:

Consider a particle of mass 'm' moving with velocity 'v' less than velocity of light then energy of particle is,

$$\mathbf{E} = \frac{1}{2}\mathbf{m}\mathbf{v}^2 \quad -----(1)$$

Multiply and divide by 'm' to the right side of equation (1)

$$E = \frac{1}{2} \frac{m^2 v^2}{m}$$
$$E = \frac{1}{2} \frac{P^2}{m}$$
$$P^2 = 2mE$$

Take square root of both sides

 $\mathbf{P} = \sqrt{2\mathbf{m}\mathbf{E}}$ -----(2)

We know that the de-Broglie wavelength is,

$$\lambda = \frac{h}{p}$$
-----(3)

From equation (2), equation (3) becomes,

$$\lambda = \frac{h}{\sqrt{2mE}} - \dots - (4)$$

Hence equation (4) is expression for de-Broglie wavelength of particle moving with velocity less than velocity of light.

EXPRESSION FOR DE-BROGLIE WAVELENGTH OF PARTICLE MOVING WITH VELOCITY COMPARABLE WITH VELOCITY OF LIGHT:

Consider a particle of rest mass 'm_0'. Let it is moving with velocity $v\approx c$, then its total relativistic energy is,

$$E = k + m_0 c^2$$
 -----(1)

Squaring equation (1) in both sides, we get,

$$E^2 = (k + m_0 c^2)^2$$
-----(2)

Where, k = relativistic K.E.

According to theory of relativity,

$$E^2 = p^2 c^2 + m_0^2 c^4 - - - - - (3)$$

Equate equation (2) and equation (3)

$$(\mathbf{k} + \mathbf{m}_0 \mathbf{c}^2)^2 = \mathbf{p}^2 \mathbf{c}^2 + \mathbf{m}_0^2 \mathbf{c}^4$$
$$\mathbf{k}^2 + 2\mathbf{m}_0 \mathbf{c}^2 + \mathbf{m}_0^2 \mathbf{c}^4 = \mathbf{p}^2 \mathbf{c}^2 + \mathbf{m}_0^2 \mathbf{c}^4$$
$$\mathbf{k}^2 + 2\mathbf{m}_0 \mathbf{c}^2 = \mathbf{p}^2 \mathbf{c}^2$$
$$\mathbf{p}^2 = \frac{\mathbf{k}^2 + 2\mathbf{m}_0 \mathbf{c}^2}{\mathbf{c}^2}$$
$$\mathbf{p} = \sqrt{\frac{\mathbf{k}^2 + 2\mathbf{m}_0 \mathbf{c}^2}{\mathbf{c}^2}}$$
$$\mathbf{p} = \sqrt{\frac{\mathbf{k}^2 + 2\mathbf{m}_0 \mathbf{c}^2}{\mathbf{c}^2}} - \dots - \mathbf{(4)}$$

de-Broglie wavelength is,

$$\lambda = \frac{h}{p} - \dots - (5)$$

Put value of 'p' from equation (4) in equation (5),

$$\lambda = \frac{hc}{\sqrt{k^2 + 2m_0c^2}}$$
 -----(6)

Equation (6) is the expression for de-Broglie wavelength of particle moving with velocity comparable with velocity of light.

PHASE VELOCITY:

The velocity of plane progressive wave associated with particle in motion is called phase velocity. It is denoted by ' V_p '.

$$V_{\rm p} = \frac{\omega}{\rm k}$$

Where, $\omega = 2\pi v =$ angular frequency

$$k = \frac{2\pi}{\lambda}$$
 = wave vector

Expression For Phase Velocity:

We know that,

$$V_p = \frac{\omega}{k}$$
 -----(1)

The energy of particle behaving as wave is,

$$\mathbf{E} = \mathbf{h}\mathbf{v} \quad -----(\mathbf{2})$$

Multiply & divide by 2π at R.H.S. of equation (2)

$$\mathbf{E} = \frac{h\nu 2\pi}{2\pi}$$
$$\mathbf{E} = \hbar\boldsymbol{\omega} \quad \text{-----}(\text{since, } \boldsymbol{\omega} = 2\pi\nu \text{ and } \hbar = \frac{h}{2\pi})$$
$$\Rightarrow \boldsymbol{\omega} = \frac{E}{\hbar} \quad \text{-----}(\mathbf{3})$$

de-Broglie wavelength is,

$$\lambda = \frac{h}{p}$$

 $p = \frac{h}{\lambda}$ -----(4)

Multiply & divide by 2π at R.H.S. of equation (4)

$$\mathbf{p} = \frac{\mathbf{h}}{2\pi} \frac{2\pi}{\lambda}$$

$$\mathbf{p} = \hbar \mathbf{k} \quad \text{-----}(\text{since, } \mathbf{k} = \frac{2\pi}{\lambda} \text{ and } \hbar = \frac{\mathbf{h}}{2\pi})$$

$$\Rightarrow \mathbf{k} = \frac{\mathbf{p}}{\hbar} \quad \text{-----}(\mathbf{5})$$

Now using equation (3) & equation (5) in equation (5), we get,

$$V_{\mathbf{p}} = \frac{\frac{\mathbf{E}}{\hbar}}{\frac{\mathbf{p}}{\hbar}} = \frac{E}{\hbar} \times \frac{\hbar}{p}$$
$$V_{\mathbf{p}} = \frac{\mathbf{E}}{\mathbf{p}} - \dots - \dots - (6)$$

[i] <u>Case I</u>: If velocity of particle is less than velocity of light

i.e. v < c then energy is,

$$E = \frac{p^2}{2m}$$

Therefore equation (6) becomes,

$$V_{p} = \frac{\frac{p^{2}}{2m}}{p} = \frac{p}{2m}$$

$$V_p=\frac{mv}{2m}$$
 -----(since, p=mv)
$$V_p=\frac{v}{2}$$

Thus phase velocity is $\frac{1}{2}$ the velocity of particle.

[i] <u>Case II</u>: If velocity of particle comparable velocity of light

i.e. $v \approx c$ then energy is,

 $\mathbf{E} = \mathbf{m}\mathbf{c}^2$

Therefore equation (6) becomes,

$$V_p = \frac{mc^2}{p} = \frac{mc^2}{mv}$$
 -----(since, p=mv)
$$V_p = \frac{c^2}{v}$$

Thus phase velocity is greater than velocity of light which is impossible. This clearly shows that a single wave cannot be associated with particle in motion.

WAVE PACKET:



- It is found that particle in motion cannot be represented by a single wave to overcome this difficulty, Bohr suggested that particle in motion can be represented by a group of waves is called wave packet.
- Consider large number of progressive wave having angular frequencies in the range ω to $\omega + d\omega$ and wave vector in the range k to k + dk, super imposing one another these waves form wave

packet, the resultant consist of points of maximum & minimum amplitude.

• The group of waves associated with particle in motion such that they interfere constructively over small region around the particle is called wave packet.

GROUP VELOCITY:

The velocity with which maximum amplitude of group of waves (wave packet) moves in forward direction is called Group velocity. It is given,

$$V_g = \frac{d\omega}{dk}$$

Expression For Group Velocity:

We know that,

$$V_{g} = \frac{d\omega}{dk} - \dots - (1)$$

The energy of particle behaving as wave is,

 $E = h\nu$ -----(2)

Multiply & divide by 2π at R.H.S. of equation (2)

$$\mathbf{E} = \frac{h\nu 2\pi}{2\pi}$$

$$\mathbf{E} = \hbar \boldsymbol{\omega} \quad \text{-----}(\text{since, } \boldsymbol{\omega} = 2\pi\nu \text{ and } \hbar = \frac{h}{2\pi})$$

$$\Rightarrow \boldsymbol{\omega} = \frac{E}{\hbar}$$

$$\Rightarrow \mathbf{d}\boldsymbol{\omega} = \frac{\mathbf{d}E}{\hbar} \quad \text{------}(\mathbf{3})$$

de- Broglie wavelength is,

$$\lambda = \frac{h}{p}$$
$$p = \frac{h}{\lambda} - \dots - (4)$$

Multiply & divide by 2π at R.H.S. of equation (4)

$$p = \frac{h}{2\pi} \frac{2\pi}{\lambda}$$

$$p = \hbar k - \dots - (since, k = \frac{2\pi}{\lambda} and \hbar = \frac{h}{2\pi})$$

$$\Rightarrow k = \frac{p}{\hbar}$$

$$\Rightarrow dk = \frac{dp}{\hbar} - \dots - (5)$$

Now using equation (3) & equation (5) in equation (5), we get,

$$V_{g} = \frac{\frac{dE}{\hbar}}{\frac{dp}{\hbar}} = \frac{dE}{\hbar} \times \frac{\hbar}{dp}$$
$$V_{g} = \frac{dE}{dp} - \dots - \dots - (6)$$

[i] <u>Case I</u>: If velocity of particle is less than velocity of light

i.e. v < c then energy is,

$$E = \frac{p^2}{2m}$$

$$\Rightarrow dE = \frac{2p \, dp}{2m} = \frac{p \, dp}{m}$$

$$\frac{dE}{dp} = \frac{p}{m} = \frac{mv}{m} = v -----(since, p=mv)$$

Therefore equation (6) becomes,

$$V_{g} = \frac{dE}{dp} = v - - - - (7)$$

Thus group velocity is equal to the velocity of particle.

[i] <u>Case II</u>: If velocity of particle comparable velocity of light

i.e. $v \approx c$ then energy is,

 $E = mc^{2}$ $\Rightarrow E = pc - - - - (since \ p = mc)$ $\Rightarrow dE = dp \ c$

$$\Rightarrow \frac{\mathrm{dE}}{\mathrm{dp}} = \mathbf{c} = \mathbf{v} - \cdots - \mathbf{(8)}$$

Therefore equation (6) becomes,

$$V_g = \frac{dE}{dp} = c = v$$
-----(from 8)
 $V_g = v$

Thus group velocity is equal to velocity of particle comparable to velocity of light.

Relation between phase velocity and group velocity:

The phase velocity of wave associated with particle in motion is given by,

$$V_p = \frac{\omega}{k}$$

 $\Rightarrow \omega = V_p k$ -----(1)

The group velocity of wave packet associated with particle in motion is given by,

$$V_{g} = \frac{d\omega}{dk}$$

$$V_{g} = \frac{d}{dk} (V_{p}k)$$

$$V_{g} = V_{p} \frac{d}{dk} (k) + k \frac{d}{dk} (V_{p})$$

$$V_{g} = V_{p} + k \frac{dV_{p}}{dk} - \dots - (2)$$

We know that,

$$k = \frac{2\pi}{\lambda} \implies dk = \frac{-2\pi}{\lambda^2} d\lambda$$
-----(3)

Using equation (3) in equation (2), we get,

$$V_{g} = V_{p} + \frac{2\pi}{\lambda} \frac{dV_{p}}{\frac{-2\pi}{\lambda^{2}}d\lambda}$$
$$V_{g} = V_{p} + \frac{2\pi}{\lambda} \frac{dV_{p}}{d\lambda} (\frac{-\lambda^{2}}{2\pi})$$

$$V_{g} = V_{p} - \lambda \frac{dV_{p}}{d\lambda}$$
-----(4)

Equation (4) is the relation between phase velocity and group velocity.

<u>Case I:</u> Dispersive Medium

For dispersive medium, phase velocity is a function of wavelength i.e. different wavelength has different velocity. Therefore,

$$V_{g} = V_{p} - \lambda rac{dV_{p}}{d\lambda}$$
-----(5) $v_{g} > v_{p}$

Thus, for dispersive medium, $v_g\,$ is greater than $v_p\,.$

Case II: Non-dispersive Medium

For non-dispersive medium, $v_{\rm p}$ is not a function of wavelength i.e. all the wavelengths have same phase velocity. Therefore,

$$\frac{\mathrm{d} \mathbf{V}_{\mathrm{p}}}{\mathrm{d} \lambda} = \mathbf{0}$$

Hence equation (4) becomes,

 $V_g = V_p$ -----(from equation 6)

Thus for non-dispersive medium v_g is equal to $v_p. \label{eq:vg}$

DAVISSON AND GERMER EXPERIMENT:

It gives the experimental verification of de-Broglie hypothesis.

Construction:

- i. It consist of Tungsten filament 'F'.
- ii. When current is passed through the filament then it emits electron.



- iii. 'P' is a metal plate having small hole. It is at positive potential with respect to filament. A well collimated electron beam is obtained through hole 'S₁'.
- iv. The beam is incident on nickel crystal and is scattered in all possible direction through angle 'O' is received by the chamber.
- v. The front and back face of chamber are insulated and retarding potential is applied between them.
- vi. The chambers can move for different angles.
- vii. The electron current is measured by sensitive galvanometer.
- viii. The whole apparatus is kept in an evacuated chamber.

<u>Working:</u>

- i. The known potential difference is applied between filament and plate.
- ii. The electron beam scattered at angle 'θ' is received by chamber and corresponding electron current is measured.
- iii. The chamber is set for different angles and corresponding currents are measured.
- iv. Now apply another potential difference between filament and plate and repeat the procedure.
- v. The graph between intensity and scattering angle is plotted as below:



- vi. Thus the refraction takes place and maximum current is obtained at θ =50° and V=54 v.
- vii. According to diffraction condition of Bragg's law,

$$2dsin\theta = n\lambda$$

Here, n=1, θ =50° and 2d=2.15 Å
 $\lambda = 2.15 \times sin (50°)$
 $\lambda = 1.65 \text{ Å}$ -----(1)

Equation (1) is experimental value

viii. According to de-Broglie's hypothesis,

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = rac{\mathrm{h}}{\sqrt{2\mathrm{meV}}}$$
-----(E=eV)

Putting values of constant V=54v, m= 9.1093837015 × 10⁻³¹ kg,

 $e = 1.602 \times 10^{-19}$ coulombs, we get,

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1093837015 \times 10^{-31} \times 1.602 \times 10^{-19} \times 54}}$$

Equation (2) is theoretical value

Thus experimental and theoretical value agrees with each other.

UNCERTAINITY PRINCIPLE FOR POSITION AND MOMENTUM:

Statement: "The product of uncertainties in simultaneous determination

of position and momentum is always greater than or equal to

$$\frac{h}{2\pi}$$
 or \hbar ."
i.e. $\triangle x . \triangle P_x \ge$

Proof:

Consider a particle moving in x-direction then the wavelength of de-Broglie wave associated with particle is given by

$$\lambda = \frac{h}{p}$$
$$\Rightarrow P_{x} = \frac{h}{\lambda}$$
(1)

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Multiply and divide by 2π to equation (1) at R.H.S., we get,

$$P_{x} = \frac{h}{2\pi} \frac{2\pi}{\lambda}$$

$$P_{x} = \hbar. \text{ k-----(since, } \hbar = \frac{h}{2\pi} \text{ and } k = \frac{2\pi}{\lambda}\text{)}$$

Therefore uncertainty in momentum is,

 $P_x = \hbar \triangle k$ -----(2)

The particle in motion is represented by wave packet is the superposition of large number of waves of different wavelength. The width of wave packet is the uncertainty in the position of particle and it is given by